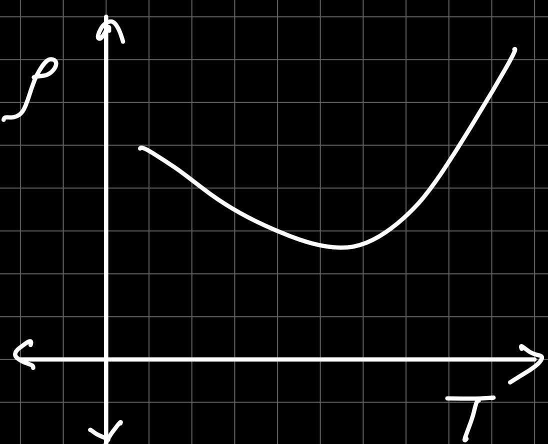
Physics 211C: Solid State Physics

Instructor: Prof. Tarun Grover   Lecture 8

Topic: Kondo effect phenomena, Anderson Impurity model and its mean-field solution

$\rho_{phonon} \sim T^5$ $\rho_{e-e} \sim T^2$ for an ordinary (pure) metal

However, when magnetic impurities are present, ρ is non-monotonic.



This motivated studies on the so called "Kondo effect"

$$H_{eff} = \sum \epsilon_k c_{k\sigma}^\dagger c_{k\sigma} + J_K \underbrace{(c^\dagger \vec{\sigma} c) \cdot \vec{S}(r=0)}_{\text{eff scattering from impurity}}$$

"Kondo model"

$$\frac{dJ_K}{dl} = J_K^2$$

↳ becomes pronounced at low Temp.

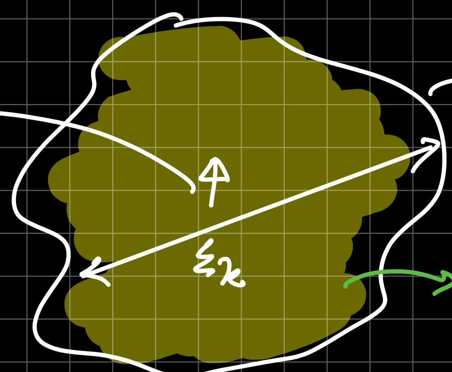
this kind of behaviour leads to $\Delta\rho \sim \log\left(\frac{T_K}{T}\right)$

$\therefore \rho \sim \log\left(\frac{T_K}{T}\right) + T^2$

has a minima

GS of Kondo model

$\epsilon_d d_\sigma^\dagger d_\sigma$



→ This entire regn forms a singlet with the ↑ spin.

"Kondo" cloud

$$|\Psi\rangle = \left[\alpha_0 + \sum_k \alpha_k c_{k\sigma} d_\sigma^\dagger\right] |\text{Filled FS}\rangle \otimes |\text{empty impurity}\rangle$$

$$\xi_K \sim e^{\frac{\#}{J_K}}$$

↳ "highly entangled state"

→ It can locally look like a FLT.

magnetic moments hybridize with CB

Anderson's Impurity model

(Ref:- Piers Coleman's book ⇒ Chapter on Kondo effect &

Philip Philip's book)

$$\mathcal{H} = \sum_k \tilde{\epsilon}_k c_{k\sigma}^\dagger c_{k\sigma} + \epsilon_d \sum d_\sigma^\dagger d_\sigma + U \underbrace{n_{d\uparrow} n_{d\downarrow}}$$

hubbard U for local moments

comes from ⇐

⇓

another gen. large trans. element material e.g. Mo

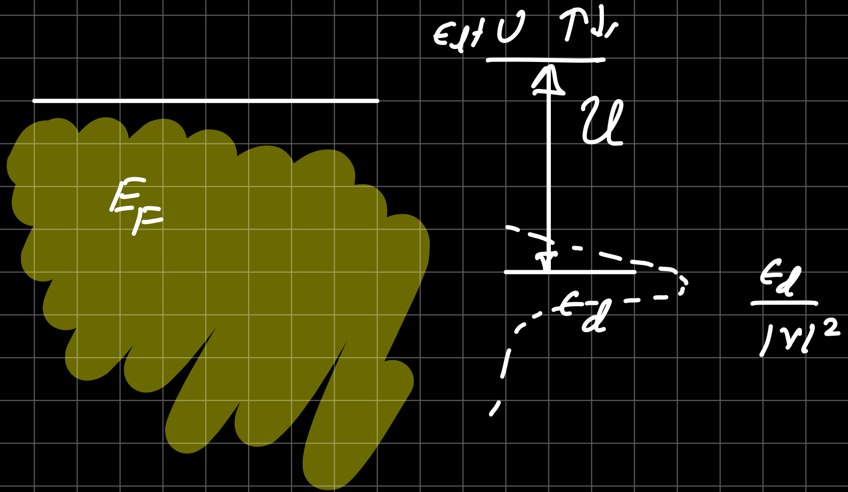
impurity bonds are generally flat

$$+ \left(\sum_k V_k c_{k\sigma}^\dagger d_\sigma + h.c.\right)$$

symmetry: $c \to c e^{i\theta}$ $d \to d e^{i\theta}$, Global $U(1)$ solution

Anderson's idea : $\langle n_{d\uparrow} \rangle = \langle c^{\dagger}_{d\uparrow} c_{d\uparrow} \rangle \neq \langle n_{d\downarrow} \rangle$

'magnetic solution'



$$U\, n_{d\uparrow} n_{d\downarrow} \xrightarrow{MF} U\left[n_{d\uparrow} \langle n_{d\downarrow} \rangle + \langle n_{d\uparrow} \rangle n_{d\downarrow} - \langle n_{d\uparrow} \rangle \langle n_{d\downarrow} \rangle \right]$$

'Green's function method for MF'

$$i \frac{\partial c_{k\sigma}}{\partial t} = [c_{k\sigma}, \mathcal{H}_{MF}] \qquad i\,\partial_t d_\sigma = [d_\sigma, \mathcal{H}_{MF}]$$

$$\omega\, c_{k\sigma} = \epsilon_k c_{k\sigma} + V_k d_\sigma \quad -① $$

$$\omega\, d_\sigma = \epsilon_d d_\sigma + U \langle n_{d-\sigma} \rangle d_\sigma + \sum_k V_k^* c_{k\sigma} \quad -② $$

From ①, $c_{k\sigma} = \dfrac{d_\sigma V_k}{\omega - \epsilon_k}$

$$\Rightarrow \omega = \epsilon_d + U \langle n_{d-\sigma} \rangle + \sum_k \frac{|V_k|^2}{\omega - \epsilon_k}$$

$$G_{dd} = \langle d_\sigma^+(\omega)\, d_\sigma(\omega) \rangle = \frac{1}{\omega - \left[\epsilon_d + U\langle n_{d-\sigma}\rangle + \sum_k \frac{|V_k|^2}{\omega - \epsilon_k}\right]}$$

dos: $\operatorname{Im} G(\omega + i\epsilon)$

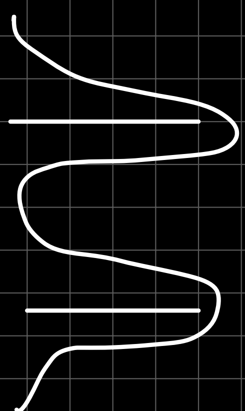
$$\rho_{d\sigma}(\omega) = \operatorname{Im} G_{dd,\sigma}(\omega + i\epsilon)\Big|_{\epsilon = 0^+}$$

$$= \frac{1}{\pi} \frac{\Gamma}{\Gamma^2 + (\omega - \epsilon_d - U\langle n_{d-\sigma}\rangle)}$$

where $\Gamma = \pi \sum_k |V_k|^2 \, \delta(\omega - \epsilon_k)$

(argument for $PV\left(\frac{1}{x}\right)$ term leading to level shifts)

$$\simeq \pi \, \overline{|V_k|}^2 \rho(E_F)$$



$$\langle n_{d\sigma} \rangle = \int_{-\infty}^{\epsilon_F} \rho_{d\sigma}(\omega)\, d\omega$$

$$= \frac{1}{\pi} \cot^{-1}\left(\frac{\tilde{\epsilon}_d + U\langle n_{d-\sigma}\rangle}{\Gamma}\right)$$

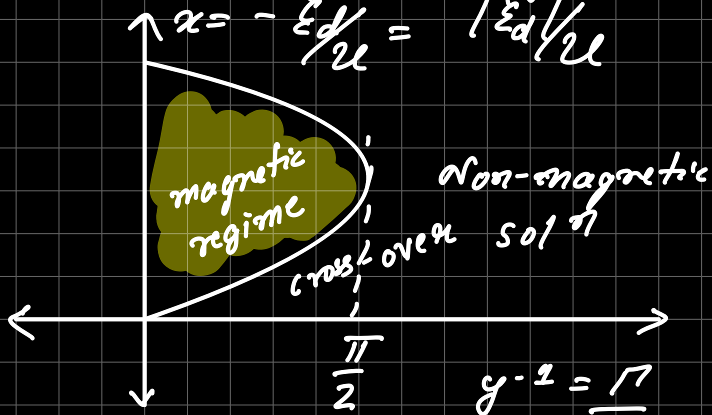
$$\tilde{\epsilon}_d = \epsilon_d - \epsilon_F$$

Limits:- ① $U \to 0$, uncoupled ↑ & ↓ spins

$$\langle n_{d\uparrow} \rangle = \langle n_{d\uparrow} \rangle$$

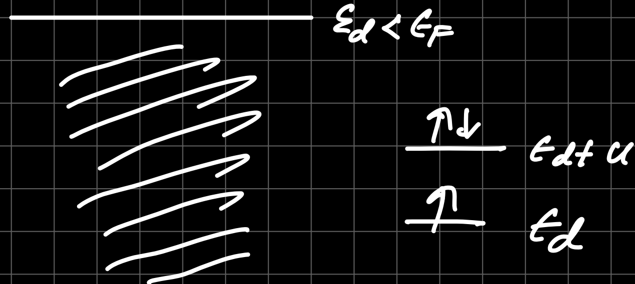
② $\Gamma \to \infty$, $\cot^{-1}(0) = \frac{\pi}{2}$ $\langle n_{d\uparrow} \rangle = \frac{1}{2} = \langle n_{d\downarrow} \rangle$

so much hybridizat^n such that it gets diffused.



misleading as SSB can't happen for a 0D Quant. system

$\tilde{\varepsilon}_d < 0$

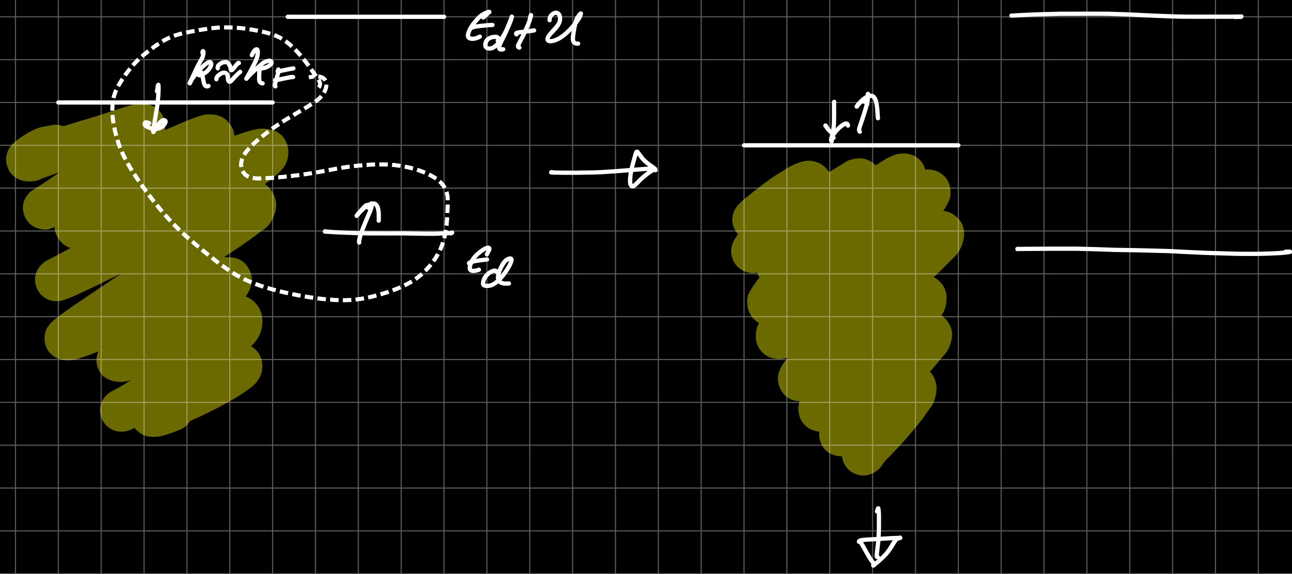


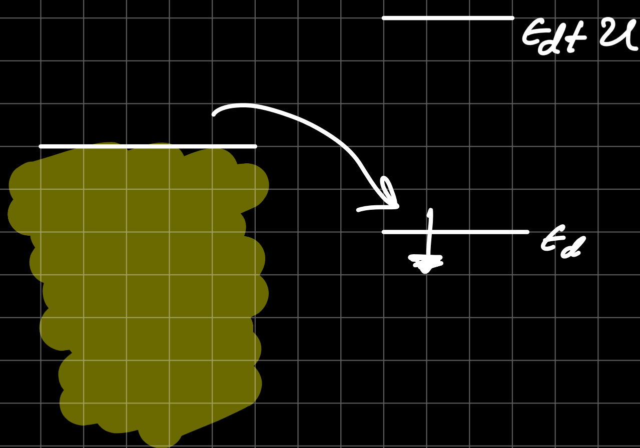
cross-over $\neq$ phase transition

<u>relevance</u>:- The magnetic regime shows that spins can strongly interact with a FL.

Variational wf^n for single impurity Anderson model

Spin fluctuation via virtual processes





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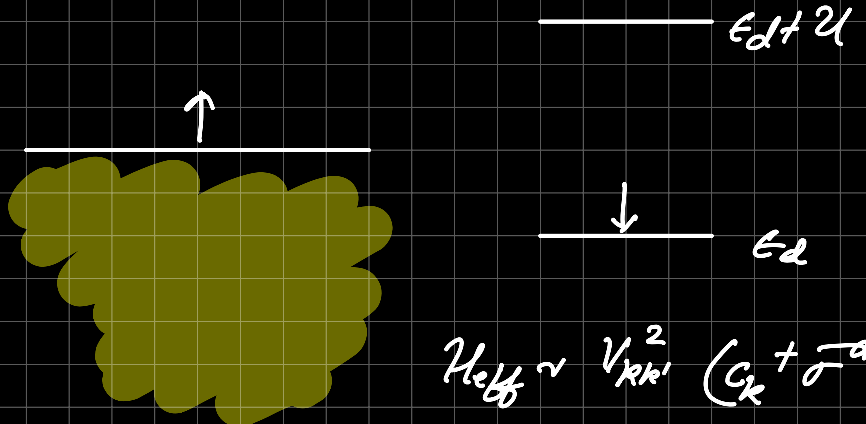
$c_\uparrow^+ \; c_\downarrow \cdot S^-$

$\Downarrow$ SU(2) symmetry

$c^\dagger \sigma c \cdot S$



$$H_{eff} \sim V_{kk'}^2 \left(c_k^\dagger \sigma^a c_{k'} \cdot S^a \right) \times \frac{1}{[\text{en. of virtual state}]}$$

$$\Delta E_1 \sim \epsilon_d - \epsilon_f$$

$$\Delta E_2 \sim \mathcal{U} + \epsilon_d - \epsilon_f$$

$$\mathcal{H}_{eff} \sim + \frac{|V|^2}{(\epsilon_d - \epsilon_f)} \sum_{kk'} c_k^{\dagger} \vec{\sigma} c_{k'} \cdot \vec{S}(0)$$

Antiferromagnetic interaction

$$\sim \underbrace{J_K \, c_{(0)}^{\dagger} \vec{\sigma} c_{(0)} \cdot \vec{S}}$$

form of Kondo model is motivated by such 2nd order processes.

Variational $W f^n$ for Kondo model

$$|\psi\rangle = \left[\alpha_0 + \sum_{k<k_F} \alpha_k^{\sigma} c_{k\sigma} d_{\sigma}^{\dagger}\right] |0\rangle$$

$$|0\rangle = |\text{Filled FS}\rangle \otimes |\text{empty d-level}\rangle$$

$$E_{var} = \frac{\langle\psi|\mathcal{H}|\psi\rangle}{\langle\psi|\psi\rangle} = \frac{2 \sum_{k<k_F} \left[\alpha_k (\epsilon_d - \epsilon_k) + 2\alpha_k V_k\right]}{\alpha_0^2 + 2\sum_k \alpha_k^2}$$

minimize

$$E_{var} = 2 \sum_{k<k_F} \frac{|V_k|^2}{E_{var} - \epsilon_d + \tilde{\epsilon}_k}$$

Binding energy $= \Delta_K = E_{var} - \epsilon_d$

when $\Delta_k < 0$, then entangled state preferred

$$\epsilon_d + \Delta_K = 2 \sum_{k<k_F} \frac{|V_k|^2}{\Delta_K + \tilde{\varepsilon}_k} \approx -2N(0) \int_{\epsilon_F}^{0} \frac{V^2 d\tilde{\varepsilon}}{\Delta_K + \tilde{\varepsilon}}$$

Kondo, not $\vec{k}$

$$\rightarrow \boxed{\Delta_K \approx -\epsilon_F \, e^{-\frac{1}{2N(0) J_K}}}$$

$\xi_K \sim \frac{v_F}{\Delta_K}$ ~ size of Kondo cloud.

$J_K \sim 1$

$\Downarrow$ RG

same as BCS (& like 1d ising)

$\frac{|V|^2}{\varepsilon_f} \sim J_K$